

The Ramsey number of generalized loose paths in uniform Hypergraphs

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Abstract

Let $H = (V, E)$ be an r -uniform hypergraph. For each $1 \leq s \leq r-1$, an s -path $\mathcal{P}_n^{r,s}$ in H of length n is a sequence of distinct vertices $v_1, v_2, \dots, v_{s+n(r-s)}$ such that $\{v_{1+i(r-s)}, \dots, v_{s+(i+1)(r-s)}\} \in E(H)$ for each $0 \leq i \leq n-1$. Recently, the Ramsey number of 1-paths and 1-cycles in uniform hypergraphs attracted a lot of attention. The asymptotic value of $R(\mathcal{P}_n^{3,1}, \mathcal{P}_n^{3,1})$ was first determined. The exact values of $R(\mathcal{P}_3^{r,1}, \mathcal{P}_3^{r,1})$ and $R(\mathcal{P}_4^{r,1}, \mathcal{P}_4^{r,1})$ are known; for $n \geq m \geq 1$, $R(\mathcal{P}_n^{3,1}, \mathcal{P}_m^{3,1})$, $R(\mathcal{P}_n^{3,1}, \mathcal{C}_m^{3,1})$, and $R(\mathcal{C}_n^{3,1}, \mathcal{C}_m^{3,1})$ are also proved. In this paper, we investigate the Ramsey number of $r/2$ -paths for even r . We prove the following exact results:

$$R(\mathcal{P}_n^{r,r/2}, \mathcal{P}_3^{r,r/2}) = \frac{(n+1)r}{2} + 1 \quad \text{and} \quad R(\mathcal{P}_n^{r,r/2}, \mathcal{P}_4^{r,r/2}) = \frac{(n+1)r}{2} + 1.$$

All approaches dealing with 1-path can not be applied to the studying of $r/2$ -path for even r . The main ingredients of the proofs are the parity of different types of edges and the analysis how does the color of one type of edges forces the color of the other type of edges.

1 Introduction

An r -uniform hypergraph H is a pair $H = (V, E)$, where V is a set of vertices and E is a collection of r -subsets of V . For two r -uniform hypergraphs H_1 and H_2 , the *Ramsey number* $R(H_1, H_2)$ is the minimum value of N such that each red-blue coloring of edges in the complete r -uniform hypergraph K_N^r on N vertices contains either a red H_1 or a blue H_2 . Let H be an r -uniform hypergraph. For each $1 \leq s \leq r-1$, an s -path $\mathcal{P}_n^{r,s}$ in H with length n is a sequence of distinct vertices $v_1, v_2, \dots, v_{s+n(r-s)}$ such that $\{v_{1+i(r-s)}, \dots, v_{s+(i+1)(r-s)}\}$ is an edge of H for each $0 \leq i \leq n-1$. Similarly, an s -cycle $\mathcal{C}_n^{r,s}$ of length n is a sequence of vertex $v_1, v_2, \dots, v_{s+n(r-s)}$ such that $\{v_{1+i(r-s)}, \dots, v_{s+(i+1)(r-s)}\}$ is an edge of H for each $0 \leq i \leq n-1$, $v_1, \dots, v_{n(r-s)}$ are distinct, and $v_{n(r-s)+j} = v_j$ for each $1 \leq j \leq s$. An s -path (and an s -cycle) is *loose* if $1 \leq s \leq r/2$ and an s -path (and an s -cycle) is *tight* if $r/2 < s \leq r-1$.

When $r = 2$ and $s = 1$, we get the definition of paths and cycles in graphs. A classical result from Ramsey theory [3] says $R(P_n, P_m) = n + \lfloor \frac{m+1}{2} \rfloor$ for $n \geq m \geq 1$; it is also known [1, 2] that $R(P_n, C_m) = R(P_n, P_m) = n + \frac{m}{2}$ for $n \geq m$ and m even. One may ask what is the Ramsey number of paths and cycles in uniform hypergraphs?

The following construction [6] was used to show a lower bound on $R(\mathcal{P}_n^{3,1}, \mathcal{P}_n^{3,1})$ for $n \geq 1$; we can adapt it to show that $N = s + n(r-s) + \lfloor \frac{m+1}{2} \rfloor - 2$ is a lower bound on $R(\mathcal{P}_n^{r,s}, \mathcal{P}_m^{r,s})$

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for $n \geq m \geq 1$ and $1 \leq s \leq r-1$. To see this, we partition the vertex set of K_N^r into two subsets A and B , where $|A| = s + n(r-s) - 1$ and $|B| = \lfloor \frac{m+1}{2} \rfloor - 1$; we color all edges f satisfying $V(f) \subseteq A$ or $V(f) \subseteq B$ red and the remaining edges blue. Observe that the number of vertices in an s -path with length n equals $s + n(r-s)$, so there is no red $\mathcal{P}_n^{r,s}$. Obviously, there is also no blue s -path of length m and we proved the lower bound.

We have the following interesting question which asks whether the construction above gives the true values of $R(\mathcal{P}_n^{r,s}, \mathcal{P}_m^{r,s})$.

Question 1 *Is $R(\mathcal{P}_n^{r,s}, \mathcal{P}_m^{r,s}) = s + n(r-s) + \lfloor \frac{m+1}{2} \rfloor - 1$ for $n \geq m \geq 1$ and $1 \leq s \leq r-1$?*

There are some positive answers to this question for the case $s = 1$. It was shown by Haxell et.al [6] that $R(\mathcal{P}_n^{3,1}, \mathcal{P}_n^{3,1})$, $R(\mathcal{C}_n^{3,1}, \mathcal{C}_n^{3,1})$, and $R(\mathcal{P}_n^{3,1}, \mathcal{C}_n^{3,1})$, equal $\frac{5n}{2}$ asymptotically. Later, Gyárfás and Raeisi [4] extended this result to all $r \geq 3$; namely they proved that $R(\mathcal{P}_n^{r,1}, \mathcal{P}_n^{r,1})$, $R(\mathcal{P}_n^{r,1}, \mathcal{C}_n^{r,1})$, and $R(\mathcal{C}_n^{r,1}, \mathcal{C}_n^{r,1})$ are asymptotically equal to $\frac{(2r-1)n}{2}$. There are some exact results on short paths and cycles. Gyárfás, Sárközy, and Szemerédi [5] first showed

$$R(\mathcal{P}_3^{r,1}, \mathcal{P}_3^{r,1}) = R(\mathcal{P}_3^{r,1}, \mathcal{C}_3^{r,1}) = R(\mathcal{C}_3^{r,1}, \mathcal{C}_3^{r,1}) + 1 = 3r - 1;$$

they also proved

$$R(\mathcal{P}_4^{r,1}, \mathcal{P}_4^{r,1}) = R(\mathcal{P}_4^{r,1}, \mathcal{C}_4^{r,1}) = R(\mathcal{C}_4^{r,1}, \mathcal{C}_4^{r,1}) + 1 = 4r - 2.$$

For $r = 3$ and $s = 1$, the exact value of long path is determined. Maherani et.al [8] first proved for $n \geq \lfloor \frac{5m}{4} \rfloor$, we have

$$R(\mathcal{P}_n^{3,1}, \mathcal{P}_m^{3,1}) = 2n + \lfloor \frac{m+1}{2} \rfloor.$$

Recently, Meherani and Shahsiah [9] showed for $n \geq m \geq 1$, we have

$$R(\mathcal{P}_n^{3,1}, \mathcal{P}_m^{3,1}) = R(\mathcal{P}_n^{3,1}, \mathcal{C}_m^{3,1}) = R(\mathcal{C}_n^{3,1}, \mathcal{C}_m^{3,1}) + 1 = 2n + \lfloor \frac{m+1}{2} \rfloor \text{ and } R(\mathcal{P}_m^{3,1}, \mathcal{C}_n^{3,1}) = 2n + \lfloor \frac{m-1}{2} \rfloor.$$

For more details on small Ramsey numbers, the reader is referred to the dynamic survey paper [10].

As the author's best knowledge, there is no attempt to study the Ramsey number of other types of paths in hypergraphs. In this paper, we will show some exact results for $s = r/2$ and r even. Before we state the theorems, we have the following lemma.

Lemma 1 *For each $s \geq 1$ and $n \geq 2$, we have*

$$R(\mathcal{P}_n^{2s,s}, \mathcal{P}_2^{2s,s}) = (n+1)s.$$

The proof of this lemma is simple and it is omitted here. We will prove the following two main theorems.

Theorem 1 *For each $s \geq 1$ and $n \geq 3$, we have*

$$R(\mathcal{P}_n^{2s,s}, \mathcal{P}_3^{2s,s}) = (n+1)s + 1.$$

Theorem 2 *For each $s \geq 1$ and $n \geq 4$, we have*

$$R(\mathcal{P}_n^{2s,s}, \mathcal{P}_4^{2s,s}) = (n+1)s + 1.$$

Notice that theorems above provide partial positive answer to Question 1 for $s = r/2$ and r even. To prove Theorem 1 and Theorem 2, we will need only to prove the upper bound.

Throughout this paper, for a red-blue coloring of a uniform hypergraph, we use \mathcal{F}_{red} (and $\mathcal{F}_{\text{blue}}$) to denote the subhypergraph induced by all red (and blue) edges respectively. Since we

will work on a fixed type path $\mathcal{P}_n^{2s,s}$ in section 2 and section 3, we will drop the superscripts and write \mathcal{P}_n for $\mathcal{P}_n^{2s,s}$. Fix an $m \geq 2$, let $f_i = \{A_i, A_{i+1}\}$ for $1 \leq i \leq m-1$. We will write f_1, f_2, \dots, f_{m-1} as an s -path of length $m-1$; we also write the path as A_1, A_2, \dots, A_m on some occasions. We will refer A_1 and A_m as ending s -sets of \mathcal{P}_n . If $g_1 = \{A_i, C, C'\}$ and $g_2 = \{C, C', A_{i+2}\}$ for some disjoint sets C and C' , then we get a new path of length $m-1$ by replacing the edges f_i and f_{i+1} by g_1 and g_2 respectively; we will write this new path as $A_1, A_2, \dots, A_i, g_1, g_2, A_{i+2}, \dots, A_m$ without arising any confusion.

The paper is organized as follows. The proof of Theorem 2 will need the truth of Theorem 1, so we will prove Theorem 1 in section 2 and Theorem 2 will be proved in section 3. We will give some concluding remarks in the last section.

2 Proof of Theorem 1

For a fixed s , Theorem 1 will be proved by induction on n . The idea for proving the base step and the inductive step are similar; we give an outline for the inductive step here. Suppose Theorem 1 holds for all $3 \leq k \leq m-1$. Let c be a red-blue coloring of edges in $K_{(m+1)s+1}$. As $(m+1)s+1 > R(\mathcal{P}_{m-1}, \mathcal{P}_3)$ by the inductive hypothesis, either there is a red \mathcal{P}_{m-1} , or there is a blue \mathcal{P}_3 . We need only to consider the former case and also assume that there is no red \mathcal{P}_m . Let $\{A_1, A_2, \dots, A_m\}$ be mutually disjoint s -sets of $[ms]$ and B be the remaining $s+1$ vertices. We fix a red path A_1, A_2, \dots, A_m and aim to find a blue \mathcal{P}_3 . For each $0 \leq l \leq s$, we say an edge f is of type $(l, s, s-l)$ if $|f \cap B| = l$, $|f \cap A_p| = s$ for some $1 \leq p \leq m$, and $|f \cap A_q| = s-l$ for some $1 \leq q \leq m$ with $q \neq p$. We will pair edges of types $(s+1-l, s, l-1)$ and $(l, s, s-l)$ as well as edges of types $(l, s, s-l)$ and $(s-l, s, l)$. Lemma 2 and Lemma 3 will show how does the color of edges of the first type forces the color of edges of the second type under some assumptions. Note m is fixed and $m \geq 3$.

Lemma 2 *Assume $\{A_i, A_j\} \subseteq \mathcal{F}_{\text{red}}$ for each $1 \leq i \neq j \leq m$ and there is no $\mathcal{P}_m \subseteq \mathcal{F}_{\text{red}}$. For each $1 \leq l \leq \lfloor \frac{s}{2} \rfloor$, if all edges of the type $(s+1-l, s, l-1)$ are blue, then the existence of a blue edge of the type $(l, s, s-l)$ implies the existence a blue \mathcal{P}_3 .*

Proof: Suppose that there is some blue edge g_1 of the type $(l, s, s-l)$. Without loss of generality, we can assume $g_1 = \{B', A_1, A'_i\}$, where B' is an l -subset of B and $A'_i \subseteq A_i$ with $|A'_i| = s-l$ for some $2 \leq i \leq m$. Choose A''_i to be an $(l-1)$ -subset of $A_i \setminus A'_i$. Since we assume $m \geq 3$, let A_j be an s -set which is different from A_1 and A_i . We define

$$g_2 = \{A_1, B \setminus B', A''_i\} \text{ and } g_3 = \{B \setminus B', A''_i, A_j\}.$$

By the assumption, both g_2 and g_3 are blue. Now g_1, g_2 , and g_3 induce a blue \mathcal{P}_3 . We proved the lemma. \square

Lemma 3 *Assume $\{A_i, A_j\} \subseteq \mathcal{F}_{\text{red}}$ for each $1 \leq i \neq j \leq m$ and there is no $\mathcal{P}_m \subseteq \mathcal{F}_{\text{red}}$. For each $1 \leq l \leq \lfloor \frac{s-1}{2} \rfloor$, if all edges of the type $(l, s, s-l)$ are red, then all edges of the type $(s-l, s, l)$ are blue.*

Proof: Suppose indirectly that there is some red edge g_1 of the type $(s-l, s, l)$. Without loss of generality, we can assume $g_1 = \{B', A_1, A'_i\}$ for some $B' \subseteq B$ with $|B'| = s-l$ and $A'_i \subseteq A_i$ with $|A'_i| = l$. Pick an arbitrary l -subset B'' of $B \setminus B'$ and define

$$g_j = \begin{cases} \{A_{j-1}, A_j\} & \text{if } 2 \leq j \leq i-1; \\ \{A_{i-1}, A_m\} & \text{if } j = i; \\ \{A_{m-(j-i-1)}, A_{m-(j-i)}\} & \text{if } i+1 \leq j \leq m-1; \\ \{A_{i+1}, A_i \setminus A'_i, B''\} & \text{if } j = m. \end{cases}$$

By the assumption, $g_j \subseteq \mathcal{F}_{\text{red}}$ for each $2 \leq j \leq m$. Now g_1, g_2, \dots, g_m form a red \mathcal{P}_m , which is a contradiction to the assumption. We completed the proof of the lemma. \square

The next lemma will tell us that the combination of two Lemmas above forces a blue \mathcal{P}_3 under the conditions.

Lemma 4 Assume $\{A_i, A_j\} \subseteq \mathcal{F}_{\text{red}}$ for each $1 \leq i \neq j \leq m$ and there is no $\mathcal{P}_m \subseteq \mathcal{F}_{\text{red}}$. Then there must be a blue \mathcal{P}_3 .

Proof: Since there is no red \mathcal{P}_m , all edges of the type $(s, s, 0)$ must be blue. We start to apply Lemma 2 and Lemma 3 alternatively. For each $1 \leq j \leq \lfloor \frac{s-1}{2} \rfloor$, we first apply Lemma 2 with $l = j$; and we stop if it succeeds to give us a blue \mathcal{P}_3 ; otherwise, we get all edges of the type $(j, s, s-j)$ are red and we apply Lemma 3 with $l = j$. If we stop for some $1 \leq j \leq \lfloor \frac{s-1}{2} \rfloor$, then we find a blue \mathcal{P}_3 and we complete the proof. Otherwise, we assume Lemma 2 fails to produce a blue \mathcal{P}_3 for each $1 \leq j \leq \lfloor \frac{s-1}{2} \rfloor$.

If s is odd, then we obtain that all edges of the type $(\frac{s+1}{2}, s, \frac{s-1}{2})$ are blue by Lemma 3 with $l = \frac{s-1}{2}$. We choose B' to be a subset of B with size $\frac{s+1}{2}$, A'_1 and A''_1 to be disjoint subsets of A_1 with size $\frac{s-1}{2}$. We define

$$g_1 = \{B', A'_1, A_2\}, \quad g_2 = \{A_2, A''_1, B \setminus B'\}, \quad \text{and} \quad g_3 = \{A'_1, B \setminus B', A_3\}.$$

We get a blue \mathcal{P}_3 with edges g_1, g_2 , and g_3 .

Similarly, if s is even, then we get that all edges of the type $(\lceil \frac{s+1}{2} \rceil, s, \lfloor \frac{s-1}{2} \rfloor)$ are blue by Lemma 3 with $l = \lfloor \frac{s-1}{2} \rfloor$. We need to apply Lemma 2 with $l = \lceil \frac{s-1}{2} \rceil$ (note $\lceil \frac{s-1}{2} \rceil = \frac{s}{2}$) again. If Lemma 2 does not yield a blue \mathcal{P}_3 , then all edges of the type $(\frac{s}{2}, s, \frac{s}{2})$ are red. We pick two disjoint subsets B' and B'' from B with size $\frac{s}{2}$, and a subset A'_1 from A_1 with size $\frac{s}{2}$. We define

$$g_j = \begin{cases} \{B', A'_1, A_2\} & \text{if } j = 1; \\ \{A_j, A_{j+1}\} & \text{if } 2 \leq j \leq m-1; \\ \{A_m, A_1 \setminus A'_1, B''\} & \text{if } j = m. \end{cases}$$

Clearly, g_j is red for each $1 \leq j \leq m$ and we obtain a red \mathcal{P}_m , which is a contradiction to the initial assumption. Therefore, there must be a blue \mathcal{P}_3 when we apply Lemma 2 with $l = \lceil \frac{s-1}{2} \rceil$. We finished the proof of the lemma. \square

With all lemmas in hand, we are ready to prove Theorem 1.

Proof of Theorem 1: We will prove the theorem by induction on n . The base step is $n = 3$. Let c be a 2-coloring of K_{4s+1} . Since $4s+1 \geq R(\mathcal{P}_3, \mathcal{P}_2)$, either there is some red \mathcal{P}_3 , or there is some blue \mathcal{P}_2 . If we are in the previous case, then there is nothing to show. Thus we assume a maximum blue path is A_1, A_2, A_3 . Let B be the remaining $s+1$ vertices. Observe that the edges $\{B', A_1\}$ and $\{B', A_3\}$ must be red for each s -subset B' of B . If $\{A_1, A_3\}$ is a blue edge, then a red \mathcal{P}_3 follows from Lemma 4 by swapping colors. If $\{A_1, A_3\}$ is red, then $\{B', A_1\}, \{A_1, A_3\}, \{A_3, B'\}$ be a red \mathcal{C}_3 for some $B' \subseteq B$. If there is no red \mathcal{P}_3 , then there has to be a blue \mathcal{P}_3 by Lemma 4, which is a contradiction. In either case, we are able to find a red \mathcal{P}_3 and we completed the proof for the base step.

Assume Theorem 1 holds for all $3 \leq k \leq m-1$ with $m \geq 4$. Consider a 2-coloring c of $K_{(m+1)s+1}$. Since $(m+1)s+1 \geq R(\mathcal{P}_{m-1}, \mathcal{P}_3) = ms+1$ by the inductive assumption, either there is a red \mathcal{P}_{m-1} , or there is a blue \mathcal{P}_3 . We need only to consider the case that the maximum length of a red path is $m-1$. Let f_1, f_2, \dots, f_{m-1} be a red \mathcal{P}_{m-1} , where $f_i = \{A_i, A_{i+1}\}$ for $1 \leq i \leq m-1$. Let B be the remaining $s+1$ vertices. Since there is no red \mathcal{P}_m , the edges $\{B', A_1\}$ and $\{B', A_m\}$ must be blue for each subset B' of B with size s . We have the following mutually disjoint cases.

Case 1: Either $\{A_1, A_j\} \subseteq \mathcal{F}_{\text{blue}}$ for some $3 \leq j \leq m-1$ or $\{A_k, A_m\} \subseteq \mathcal{F}_{\text{blue}}$ for some $2 \leq k \leq m-2$. We observe that $\{A_m, B'\}, \{B', A_1\}, \{A_1, A_j\}$ form a blue \mathcal{P}_3 in the previous case, and $\{A_1, B'\}, \{B', A_m\}, \{A_m, A_k\}$ form a blue \mathcal{P}_3 in the later case.

Case 2: We have $\{A_1, A_i\} \subseteq \mathcal{F}_{\text{red}}$ for each $3 \leq i \leq m-1$ and $\{A_i, A_m\} \subseteq \mathcal{F}_{\text{red}}$ for each $2 \leq i \leq m-2$. Moreover, there are $2 \leq j < k \leq m-1$ such that $\{A_j, A_k\} \subseteq \mathcal{F}_{\text{blue}}$. We define

$$g_q = \begin{cases} f_q & \text{if } 1 \leq q \leq k-2; \\ \{A_{k-1}, A_m\} & \text{if } q = k-1; \\ \{A_{m-(q-k)}, A_{m-(q-k)-1}\} & \text{if } k \leq q \leq m-1. \end{cases}$$

Notice that g_1, g_2, \dots, g_{m-1} is a new red \mathcal{P}_{m-1} . Now A_k is an ending s -set of this new path and we can find a blue \mathcal{P}_3 by the same way as Case 1.

Case 3: We have $\{A_i, A_j\} \subseteq \mathcal{F}_{\text{red}}$ for all $1 \leq i \neq j \leq m$ such that $\{i, j\} \neq \{1, m\}$. Now if $\{A_1, A_m\}$ is blue, then we can find a blue \mathcal{P}_3 by the same argument as Case 2; namely by finding a new red \mathcal{P}_{m-1} with one of A_1 and A_m as an ending s -set but not the other one. If $\{A_1, A_m\} \subseteq \mathcal{F}_{\text{red}}$, then a blue \mathcal{P}_3 is ensured by Lemma 4.

We finished the proof of the inductive step and completed the proof the theorem. \square

3 Proof of Theorem 2

For a fixed $s \geq 1$, we will also prove Theorem 2 by induction on n . The main work is on the inductive step. We assume $R(\mathcal{P}_k, \mathcal{P}_4) = (k+1)s+1$ for all $4 \leq k \leq m-1$. For the inductive step, let c be a red-blue coloring of edges of $K_{(m+1)s+1}$. Since $(m+1)s+1 \geq R(\mathcal{P}_{m-1}, \mathcal{P}_4) = ms+1$ by the inductive hypothesis, either there is some red \mathcal{P}_{m-1} or there is some blue \mathcal{P}_4 . There is nothing to show if either there is some $\mathcal{P}_m \subseteq \mathcal{F}_{\text{red}}$ or a $\mathcal{P}_4 \subseteq \mathcal{F}_{\text{blue}}$. Thus we assume that the maximum length of a red path is $m-1$; our goal is to find a blue \mathcal{P}_4 under this condition. Let A_1, A_2, \dots, A_m be a fixed red \mathcal{P}_{m-1} induced by c , where $\{A_1, A_2, \dots, A_m\}$ is a collection of mutually disjoint s -sets of $[ms]$. Let $B = [(m+1)s+1] \setminus [ms]$. We will frequently replace some edges of the existed red \mathcal{P}_{m-1} to obtain a new red \mathcal{P}_{m-1} with new ending s -sets. To get a blue \mathcal{P}_4 , a blue edge f with vertices from $\cup_{i=1}^m A_i$ will help us a lot. There are many possible arrangements of the vertices of f . The simplest case is $f = \{A_i, A_j\}$ for some $1 \leq i \neq j \leq m$; we will show that we can always reduce the case $f = \{A_i, A_j\}$ to the case $f = \{A_1, A_p\}$ for some $3 \leq p \leq m-1$. If $f = \{A_1, A_p\} \subseteq \mathcal{F}_{\text{red}}$, then the following lemmas tells us how can we find the desired blue \mathcal{P}_4 under some conditions. We will frequently utilize the following fact.

Fact 1 *Let A_1, \dots, A_m be a maximum red \mathcal{P}_{m-1} induced by c . Then $\{A_1, B'\}, \{A_m, B'\} \subseteq \mathcal{F}_{\text{blue}}$ for each s -subset B' of B .*

The fact follows from the maximality of the red path \mathcal{P}_{m-1} .

Fix a fixed red path A_1, A_2, \dots, A_m , we say an edge f is of type $(l, s, s-l)$ if $|f \cap B| = l$, $|f \cap A_j| = s$ for some $1 \leq j \leq m$ with $j \neq 2$, and $|f \cap A_2| = s-l$. Lemma 5 to Lemma 7 play the same role as Lemma 2 to Lemma 4.

Lemma 5 *Assume $\{A_1, A_p\} \subseteq \mathcal{F}_{\text{blue}}$ for some $3 \leq p \leq m-1$, $\{A_1, A_i\} \subseteq \mathcal{F}_{\text{red}}$ for $3 \leq i \neq p \leq m-1$, and $\{A_j, A_m\} \subseteq \mathcal{F}_{\text{red}}$ for $2 \leq j \leq m-2$. Furthermore, there is no red \mathcal{P}_m . Fix $1 \leq l \leq \lfloor \frac{s}{2} \rfloor$. If all edges of the type $(s+1-l, s, l-1)$ are blue, then the existence of a blue edge of the type $(l, s, s-l)$ implies the existence of a blue \mathcal{P}_4 .*

Proof: We assume that there is some blue edge of the type $(l, s, s-l)$, say $g_1 = \{B', A_j, A'_2\}$, where $j \neq 2$, B' is an l -subset of B , and A'_2 is an $(s-l)$ -subset of A_2 . We define A''_2 to be an arbitrary $(l-1)$ -subset of $A_2 \setminus A'_2$. We have two cases.

Case 1: $j \in \{1, p\}$. Without loss of generality, we assume $j = p$. We define

$$g_2 = \{A_p, A_1\}, \quad g_3 = \{A_1, B \setminus B', A_2''\}, \quad \text{and} \quad g_4 = \{B \setminus B', A_2'', A_m\}.$$

By the assumption, we get that g_1, g_2, g_3 , and g_4 form a blue \mathcal{P}_4 . We can obtain a blue \mathcal{P}_4 similarly when $j = 1$.

Case 2: $j \notin \{1, p\}$. We define

$$g_2 = \{A_j, B \setminus B', A_2''\}, \quad g_3 = \{B \setminus B', A_2'', A_1\}, \quad \text{and} \quad g_4 = \{A_1, A_p\}.$$

By the assumption, we obtain a blue \mathcal{P}_4 with edges g_1, g_2, g_3 , and g_4 .

We proved the lemma. \square

We also have the following lemma which is similar to Lemma 3.

Lemma 6 *Assume $\{A_1, A_p\} \subseteq \mathcal{F}_{\text{blue}}$ for some $3 \leq p \leq m-1$, $\{A_1, A_i\} \subseteq \mathcal{F}_{\text{red}}$ for $3 \leq i \neq p \leq m-1$, and $\{A_j, A_m\} \subseteq \mathcal{F}_{\text{red}}$ for $2 \leq j \leq m-2$. Furthermore, there is no red \mathcal{P}_m . Fix $1 \leq l \leq \lfloor \frac{s-1}{2} \rfloor$. If all edges of the type $(l, s, s-l)$ are red, then all edges of the type $(s-l, s, l)$ are blue.*

Proof: Suppose that there is some red edge g_1 of the type $(s-l, s, l)$. We can assume $g_1 = \{B', A_j, A_2'\}$ for some $j \neq 2$, where B' is a subset of B with size $s-l$ and A_2' is a subset of A_2 with size l . We first assume $j \neq 1$. Let B'' be an l -subset of $B \setminus B'$. We define

$$g_q = \begin{cases} \{A_{j+q-2}, A_{j+q-1}\} & \text{if } 2 \leq q \leq m-j+1; \\ \{A_m, A_{j-1}\} & \text{if } q = m-j+2; \\ \{A_{m-q+2}, A_{m-q-1}\} & \text{if } m-j+3 \leq q \leq m-2; \\ \{A_3, B'', A_2 \setminus A_2'\} & \text{if } q = m-1; \\ \{B'', A_2 \setminus A_2', A_1\} & \text{if } q = m. \end{cases}$$

Observe that g_1, \dots, g_m induce a red \mathcal{P}_m , which is a contradiction to the assumption. For $j = 1$, we can find a red \mathcal{P}_m similarly. Therefore, all edges of the type $(s-l, s, l)$ must be blue and we completed the proof of the lemma. \square

The next lemma show how can we get a blue \mathcal{P}_4 under conditions above.

Lemma 7 *Assume $\{A_1, A_p\} \subseteq \mathcal{F}_{\text{blue}}$ for some $3 \leq p \leq m-1$, $\{A_1, A_i\} \subseteq \mathcal{F}_{\text{red}}$ for $3 \leq i \neq p \leq m-1$, and $\{A_j, A_m\} \subseteq \mathcal{F}_{\text{red}}$ for $2 \leq j \leq m-2$. Furthermore, there is no red \mathcal{P}_m . Then there must be a blue \mathcal{P}_4 .*

Proof: The proof of this lemma uses the same idea as the one in the proof of Lemma 4. For each $1 \leq j \leq \lfloor \frac{s-1}{2} \rfloor$, we first apply Lemma 5 with $l = j$; if Lemma 5 succeeds to give us a blue \mathcal{P}_4 , then we stop. Otherwise, we get all edges of the type $(j, s, s-j)$ are red and we apply Lemma 6 with $l = j$. Thus, we need only to take care of the case where Lemma 5 fails to give a blue \mathcal{P}_4 for each $1 \leq j \leq \lfloor \frac{s-1}{2} \rfloor$.

If s is odd, then all edges of the type $(\frac{s+1}{2}, s, \frac{s-1}{2})$ are blue followed from Lemma 6 with $l = \frac{s-1}{2}$. Let $B' \subseteq B$ with $|B'| = \frac{s+1}{2}$, A_2' and A_2'' be two disjoint subsets of A_2 with size $\frac{s-1}{2}$. We define

$$g_1 = \{A_p, A_1\}, \quad g_2 = \{A_1, B', A_2'\}, \quad g_3 = \{B', A_2', A_m\}, \quad \text{and} \quad g_4 = \{A_m, A_2'', B/B'\}.$$

Now, we observe that g_1, g_2, g_3 , and g_4 form a blue \mathcal{P}_4 .

If s is even, then we have that all edges of the type $(\lceil \frac{s+1}{2} \rceil, s, \lfloor \frac{s-1}{2} \rfloor)$ are blue by Lemma 6 with $l = \lfloor \frac{s-1}{2} \rfloor$. Now, we appeal to Lemma 5 with $l = \lceil \frac{s-1}{2} \rceil$ (note $\lceil \frac{s-1}{2} \rceil = \frac{s}{2}$). If Lemma

5 can not give us a blue \mathcal{P}_4 , then all edges of the type $(\frac{s}{2}, s, \frac{s}{2})$ are red. Choose two disjoint subsets $B', B'' \subseteq B$ with size $\frac{s}{2}$ and a subset $A'_2 \subseteq A_2$ with size $\frac{s}{2}$. We define

$$g_j = \begin{cases} \{A_1, B', A'_2\} & \text{if } j = 1; \\ \{B', A'_2, A_3\} & \text{if } j = 2; \\ \{A_j, A_{j+1}\} & \text{if } 3 \leq j \leq m-1; \\ \{A_m, B'', A_2 \setminus A'_2\} & \text{if } j = m. \end{cases}$$

Clearly, g_1, \dots, g_m form a red \mathcal{P}_m , which is a contradiction to the assumption. Therefore, when we apply Lemma 5 with $l = \lceil \frac{s-1}{2} \rceil$, it must produce a blue \mathcal{P}_4 . We completed the proof of this lemma. \square

We have the following remark on the case $m = 5$.

Remark 1 *If $m = 5$, $\{A_1, A_3\}, \{A_3, A_5\} \subseteq \mathcal{F}_{\text{blue}}$, and $\{A_1, A_4\}, \{A_2, A_5\} \subseteq \mathcal{F}_{\text{red}}$, then there is a blue \mathcal{P}_4 .*

The proof of this remark follows exactly the same lines as Lemma 4 to Lemma 6 and it is omitted here.

As we mentioned before, a blue edge $f = \{A_i, A_j\}$ is helpful for finding a blue \mathcal{P}_4 . The next lemma will show the case $f = \{A_1, A_p\}$ for some $3 \leq p \leq m-1$.

Lemma 8 *If $\{A_1, A_p\}$ is blue for some $3 \leq p \leq m-1$, then there is a blue \mathcal{P}_4 .*

Proof: If there is some $2 \leq j \neq p \leq m-2$ such that $\{A_j, A_m\}$ is blue, then let $g_3 = \{A_1, B'\}$ and $g_4 = \{B', A_m\}$, where B' is an s -subset of B . Fact 1 implies that both g_3 and g_4 are blue. Note that $f, g_3, g_4, \{A_m, A_j\}$ form a blue \mathcal{P}_4 . In the remaining proof, we assume $\{A_j, A_m\}$ is blue for each $2 \leq j \neq p \leq m-2$. Note that the above argument gives us the assumptions in Lemma 7 for $m = 4$; thus a desired blue \mathcal{P}_4 is ensured by Lemma 7 for $m = 4$.

We leave the case $m = 5$ for a while. For $m \geq 6$, we get that either $p-1 \geq 3$ or $m-p \geq 3$. The main idea is that we find a new red path with length $m-1$ which contains A_q as an ending s -set for some $q \notin \{1, p, m\}$. We assume $p-1 \geq 3$ and the case $m-p \geq 3$ can be proved similarly. If $\{A_p, A_m\} \subseteq \mathcal{F}_{\text{blue}}$, then we consider a new red path $A_1, A_2, A_m, A_3, \dots, A_p, \dots, A_{m-1}$. Fact 1 implies $\{A_1, B'\}, \{A_{m-1}, B'\} \subseteq \mathcal{F}_{\text{blue}}$ for each s -subset B' of B . Now, $A_m, A_p, A_1, B', A_{m-1}$ is a blue \mathcal{P}_4 . Thus, we can assume $\{A_p, A_m\} \subseteq \mathcal{F}_{\text{red}}$. By the same argument, we can also assume $\{A_1, A_j\} \subseteq \mathcal{F}_{\text{red}}$ for each $3 \leq j \neq p \leq m-1$; otherwise we can find a blue \mathcal{P}_4 easily. Under the assumptions above, Lemma 7 gives a desired blue \mathcal{P}_4 .

We are left to prove the case $m = 5$. If either $p = 4$ or $p = 3$ and $\{A_3, A_5\} \in \mathcal{F}_{\text{red}}$, a blue \mathcal{P}_4 is given by Lemma 7. If $p = 3$ and $\{A_3, A_5\} \in \mathcal{F}_{\text{blue}}$, then a blue \mathcal{P}_4 is given by Remark 1. We proved the lemma. \square

The next lemma will tell us that we can reduce the general case $f = \{A_i, A_j\}$ to the case $f = \{A_1, A_p\}$.

Lemma 9 *If there is some blue edge $f = \{A_i, A_j\}$ for some $1 \leq i \neq j \leq m$, then there is a blue \mathcal{P}_4 .*

Proof: We have the following mutually disjoint cases.

Case 1: $|\{i, j\} \cap \{1, m\}| = 1$. Note that the case $f = \{A_j, A_m\}$ is the same as $f = \{A_1, A_p\}$ by the symmetry, so the case is proved by Lemma 8.

Case 2: $2 \leq i < j \leq m$ and $\{A_p, A_q\} \subseteq \mathcal{F}_{\text{red}}$ for all $|\{p, q\} \cap \{1, m\}| = 1$. We observe that $A_j, \dots, A_m, A_{j-1}, \dots, A_i, \dots, A_1$ is a new red \mathcal{P}_{m-1} and we can reduce it to Case 1.

Case 3: $\{i, j\} = \{1, m\}$ and $\{A_p, A_q\} \subseteq \mathcal{F}_{\text{red}}$ for all $\{p, q\} \neq \{1, m\}$. We form a new blue \mathcal{P}_{m-1} which is $A_1, A_2, A_m, \dots, A_3$, and we reduce it to Case 1.

We finished the proof of the lemma. \square

We already showed how a blue edge $\{A_i, A_j\}$ helps us to get a blue \mathcal{P}_4 . In general, f could intersect more than two A_i 's. The next lemma will give us a $\mathcal{P}_4 \subseteq \mathcal{F}_{\text{blue}}$ for other possible intersections between f and A_i 's. We need some notations. Given a red path $\mathcal{P}_{m-1} = A_1, A_2, \dots, A_m$ and an edge f with $V(f) \subseteq \cup_{i=1}^m A_i$, let $S(\mathcal{P}_{m-1}, f) = \{1 \leq i \leq m : f_i \cap A_i \neq \emptyset\}$. We say the fixed coloring c has **Property(i)** if the existence of some edge f and a red path \mathcal{P}_{m-1} satisfying $S(\mathcal{P}_{m-1}, f) = i$ implies that c induces a blue \mathcal{P}_4 . We have the following lemma.

Lemma 10 *For a fixed red-blue coloring c of edges of $K_{(m+1)s+1}$ without $\mathcal{P}_m \subseteq \mathcal{F}_{\text{red}}$, the coloring c has **Property(i)** for each $2 \leq i \leq \min\{m, s\}$.*

Proof: We proceed the proof by induction on i . The base step $i = 2$ is given by Lemma 9. We assume c has **Property(i)** for all $2 \leq i \leq k-1$. For the inductive step, let us fix a red \mathcal{P}_{m-1} and a blue edge f satisfying $|S(\mathcal{P}_{m-1}, f)| = k$. We can assume all edges f' with $|S(\mathcal{P}_{m-1}, f')| < k$ are red, otherwise a blue \mathcal{P}_4 is ensured by the inductive hypothesis. Without loss of generality, we assume $S(\mathcal{P}_{m-1}, f) = \{1, \dots, k\}$. Let $A'_i = f \cap A_i$ for each $1 \leq i \leq k$.

If $k \geq 4$, then we assume $|A'_1| \leq |A'_2| \leq \dots \leq |A'_k|$. Clearly, $|A'_1 \cup A'_2| \leq s$. Let C be a subset of $A_1 \cup A_2$ such that $A'_1 \cup A'_2 \subseteq C$ and $|C| = s$. Observe $\{A_3, C\} \subseteq \mathcal{F}_{\text{red}}$; otherwise, note $|S(\mathcal{P}_{m-1}, \{A_3, C\})| = 3$ and the blue \mathcal{P}_4 is given by the inductive hypothesis. Let $C' = (A_1 \cup A_2) \setminus C$ and we consider a new red path $\mathcal{P}'_{m-1} = A_m, \dots, A_3, C, C'$. Observe $|S(\mathcal{P}'_{m-1}, f)| = k-1$, we get a blue \mathcal{P}_4 by the inductive hypothesis.

If $k = 3$, then we need more argument. We need only to prove the case $|A'_i \cup A'_j| > s$ for $1 \leq i \neq j \leq 3$. We can also assume $\{A_i, A_j\} \subseteq \mathcal{F}_{\text{red}}$ for all $1 \leq i \neq j \leq m$; otherwise the base step produces a blue \mathcal{P}_4 .

We first consider that there is some $1 \leq i \leq 3$ such that $A_i \subseteq f$. Without loss of generality, we assume $A_1 \subseteq f$. Let $g_2 = \{A_2 \setminus A'_2, A_3 \setminus A'_3, A_4\}$. If $g_2 \subseteq \mathcal{F}_{\text{blue}}$, then let $g_3 = \{A_4, B'\}$ and $g_4 = \{B', A_1\}$, here $B' \subseteq B$ and $|B'| = s$. Since we can view A_4 as one of ending s -sets of a red \mathcal{P}_{m-1} , Fact 1 implies $g_3, g_4 \subseteq \mathcal{F}_{\text{blue}}$. Now, g_2, g_3, g_4, f form a blue \mathcal{P}_4 . If $g_2 \subseteq \mathcal{F}_{\text{red}}$, then we form a new red path $\mathcal{P}'_{m-1} = g_1, A_4, \dots, A_m, A_1, f$. Note $|f \cap \mathcal{P}'_{m-1}| = 2$ and the base case gives us a blue \mathcal{P}_4 . We are through in this case.

If $|A_i \cap f| < s$ for each $1 \leq i \leq 3$, then we pick a subset A''_2 from A'_2 such that $|A''_2 \cup A'_3| = s$. Let $C = (A_2 \setminus A''_2) \cup (A_3 \setminus A'_3)$. We need only to consider the case $\{A''_2, A'_3, A_4\}, \{A_1, C\} \subseteq \mathcal{F}_{\text{red}}$. If $f' = \{A''_2, A'_3, A_4\} \subseteq \mathcal{F}_{\text{blue}}$, then a blue \mathcal{P}_4 is given by the previous case by observing $|f' \cap \mathcal{P}_{m-1}| = 3$ and $A_4 \subseteq f'$. We have a similar argument for $g_2 = \{A_1, C\} \subseteq \mathcal{F}_{\text{red}}$. When $\{A''_2, A'_3, A_4\}, \{A_1, C\} \subseteq \mathcal{F}_{\text{red}}$, we observe $\mathcal{P}'_{m-1} = g_1, A_4, \dots, A_m, A_1, C$ is a new \mathcal{P}'_{m-1} , $|S(\mathcal{P}'_{m-1}, f)| = 3$, and $C \subseteq f$; we reduce this case to the previous case.

We proved the lemma. \square

We already know how to find a blue \mathcal{P}_4 if there is some blue f such that $f \subseteq \cup_{i=1}^m A_i$. Next, we assume f is red for all $f \subseteq \cup_{i=1}^m A_i$ and show how can we find a blue \mathcal{P}_4 under this assumption. We need one more definition. Fix a red path A_1, A_2, \dots, A_m and let B be the remaining $s+1$ vertices. For each $1 \leq l \leq s$, we say f is of type $(s-l, s+l)$ if $|f \cap B| = s-l$ and $|f \cap (\cup_{i=1}^m A_i)| = s+l$.

Lemma 11 *Let A_1, A_2, \dots, A_m be a red \mathcal{P}_{m-1} . Assume all edges f with $f \subseteq \cup_{i=1}^m A_i$ are red and there is no red \mathcal{P}_m . For each $1 \leq l \leq \lfloor \frac{s}{2} \rfloor$, if all edges of the type $(s-l+1, s+l-1)$ are blue, then the existence of a red edge of the type $(s-l, s+l)$ implies the existence of a blue \mathcal{P}_4 .*

Proof: Suppose that there is some edge f of the type $(s-l, s+l)$ is red. Without loss of generality, we assume $f = \{A_1, A'_2, B'\}$, where $A'_2 \subseteq A_2$ with $|A'_2| = l$ and $B' \subseteq B$ with $|B'| = s-l$. Let B'' be an l -subset of $B \setminus B'$. We define

$$g_1 = \{A_3, B'', A_2 \setminus A'_2\} \text{ and } g_2 = \{B'', A_2 \setminus A'_2, A_4\}.$$

We get both g_1 and g_2 are blue. Otherwise, if g_1 is red, then $g_1, A_3, \dots, A_m, A_1, f$ is a red \mathcal{P}_m , which is a contradiction to the assumption. If g_2 is red, then we can find a contradiction similarly. Let A'_2 be an $(l-1)$ -subset of A'_2 . We define

$$g_3 = \{A_1, A'_2, B \setminus B''\} \text{ and } g_4 = \{A'_2, B \setminus B'', A_3\}.$$

We get that both g_3 and g_4 are blue by the assumption. Now, g_3, g_4, g_1, g_2 is a blue \mathcal{P}_4 . We proved the lemma. \square

The next lemma will show how does Lemma 11 guarantee a blue \mathcal{P}_4 under the assumption.

Lemma 12 *Let A_1, A_2, \dots, A_m be a red \mathcal{P}_{m-1} . Assume all edge f satisfying $f \subseteq \cup_{i=1}^m A_i$ are red and there is no red \mathcal{P}_m . Then we have a blue \mathcal{P}_4 .*

Proof: There is no red \mathcal{P}_m implies all edges of the type (s, s) are blue. We start to apply Lemma 11 with $l = 1$. If there is some $1 \leq l \leq \lfloor \frac{s}{2} \rfloor$ such that Lemma 11 succeeds to give us a blue \mathcal{P}_4 , then we are through. Otherwise, Lemma 11 with $l = \lfloor \frac{s}{2} \rfloor$ tells us that all edges of the type $(s - \lfloor \frac{s}{2} \rfloor, s + \lfloor \frac{s}{2} \rfloor)$ are blue.

When s is odd. Let B' be a subset B with size $\frac{s+1}{2}$, and A'_1 and A'_2 be two disjoint subsets of A_1 with size $\frac{s-1}{2}$. We define

$$g_1 = \{A_2, A'_1, B'\}, \quad g_2 = \{A'_1, B', A_3\}, \quad g_3 = \{A_3, A'_1, B \setminus B'\}, \text{ and } g_4 = \{A'_1, B \setminus B', A_4\}.$$

It is easy to see that g_1, g_2, g_3, g_4 form a blue \mathcal{P}_4 .

When s is even. Let B' and B'' be two disjoint subsets B with size $\frac{s}{2}$ and A'_1 be subsets of A_1 with size $\frac{s}{2}$. We define

$$g_1 = \{A_2, A'_1, B'\}, \quad g_2 = \{A'_1, B', A_3\}, \quad g_3 = \{A_3, A_1 \setminus A'_1, B''\}, \text{ and } g_4 = \{A_1 \setminus A'_1, B'', A_4\}.$$

It is easy to see that g_1, g_2, g_3, g_4 form a blue \mathcal{P}_4 . In either case, we are able to find a blue \mathcal{P}_4 and we completed the proof. \square

We are now ready to prove Theorem 2.

Proof of Theorem 2: We prove the theorem by induction on n . For the base case, let c be a 2-coloring of edges in K_{5s+1} . As $5s+1 \geq R(\mathcal{P}_4, \mathcal{P}_3)$ by Theorem 1, either we have a red \mathcal{P}_4 or we have a blue \mathcal{P}_3 . There is nothing to show in the previous case. Thus we assume A_1, A_2, A_3, A_4 is a maximum blue path. If there is an red edge with vertices from $\cup_{i=1}^4 A_i$, then we have a red \mathcal{P}_4 by Lemma 10 with colors swapped. Otherwise, switch colors in Lemma 12 and it gives us a red \mathcal{P}_4 .

The inductive step is given by Lemma 10 and Lemma 12. We finished the proof of the theorem. \square

4 Concluding remarks

In this paper, we give a partial affirmative answer to Question 1 for $s = r/2$, r even, and $m \in \{3, 4\}$. However, unlike the paper [5], we are not able to determine the Ramsey number of small $r/2$ -cycles for even r . A possible reason is following. In [5], they proved the following statement. Let c be a red-blue coloring of edges in K_N^r , here $N = (r-1)n + \lfloor \frac{m+1}{2} \rfloor$. If

$\mathcal{C}_n^{r,1} \subseteq \mathcal{F}_{\text{red}}$, then either $\mathcal{P}_n^{r,1} \subseteq \mathcal{F}_{\text{red}}$ or $\mathcal{P}_m^{r,1} \subseteq \mathcal{F}_{\text{blue}}$. Also, if $\mathcal{C}_n^{r,1} \subseteq \mathcal{F}_{\text{red}}$, then either $\mathcal{P}_n^{r,1} \subseteq \mathcal{F}_{\text{red}}$ or $\mathcal{C}_m^{r,1} \subseteq \mathcal{F}_{\text{blue}}$. The statement above is a very important fact for $s = 1$; it helps to determine the values of $R(\mathcal{P}_n^{r,1}, \mathcal{P}_m^{r,1})$, $R(\mathcal{P}_n^{r,1}, \mathcal{C}_m^{r,1})$, and $R(\mathcal{C}_n^{r,1}, \mathcal{C}_m^{r,1})$. We can not prove a similar lemma for $s = r/2$ and r even since we are in short of vertices after we fix a red $\mathcal{C}_n^{r,r/2}$. It would be helpful to prove some lemma which connects $R(\mathcal{P}_n^{r,r/2}, \mathcal{P}_m^{r,r/2})$ to $R(\mathcal{C}_n^{r,r/2}, \mathcal{C}_m^{r,r/2})$.

To answer Question 1, we need to determine the exact values of the Ramsey number of each type of paths; it is very possible that we need different techniques to deal with different types of paths. The author strongly believe the following conjecture holds.

Conjecture 1 *For fixed $r \geq 2$ and $n \geq m \geq 3$, we have*

$$R(\mathcal{P}_n^{r,1}, \mathcal{P}_m^{r,1}) > R(\mathcal{P}_n^{r,2}, \mathcal{P}_m^{r,2}) > \dots > R(\mathcal{P}_n^{r,\lfloor r/2 \rfloor}, \mathcal{P}_m^{r,\lfloor r/2 \rfloor}).$$

There are many other interesting questions on Ramsey number of paths and cycles in hypergraphs. The only known results addressing the tight cycles is due to Haxell et.al [7] who proved the asymptotic value of $R(\mathcal{C}_n^{3,2}, \mathcal{C}_n^{3,2})$. A natural question is to determine the exact values of the Ramsey number of tight paths and cycles; the author has no inclination to whether the natural lower bound gives the true value of them.

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